NAG Fortran Library Routine Document

S13ACF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S13ACF returns the value of the cosine integral

$$\mathrm{Ci}(x) = \gamma + \ln x + \int_0^x \frac{\cos u - 1}{u}\,du, \quad x > 0$$

via the routine name, where γ denotes Euler's constant.

2 Specification

```
real FUNCTION S13ACF(X, IFAIL)
INTEGER              IFAIL
real                 X
```

3 Description

The routine calculates an approximate value for $\mathrm{Ci}(x)$.

For $0 < x \leq 16$ it is based on the Chebyshev expansion

$$\mathrm{Ci}(x) = \ln x + {\sum_{r=0}}' a_r T_r(t),\ t = 2\left(\frac{x}{16}\right)^2 - 1.$$

For $16 < x < x_{hi}$ where the value of x_{hi} is given in the Users' Note for your implementation,

$$\mathrm{Ci}(x) = \frac{f(x)\sin x}{x} - \frac{g(x)\cos x}{x^2}$$

where $f(x) = {\sum_{r=0}}' f_r T_r(t)$ and $g(x) = {\sum_{r=0}}' g_r T_r(t)$, $t = 2\left(\frac{16}{x}\right)^2 - 1$.

For $x \geq x_{hi}$, $\mathrm{Ci}(x) = 0$ to within the accuracy possible (see Section 7).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – ***real*** *Input*

On entry: the argument x of the function.

Constraint: X > 0.0.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

The routine has been called with an argument less than or equal to zero for which the function is not defined. The result returned is zero.

7 Accuracy

If E and ϵ are the absolute and relative errors in the result and δ is the relative error in the argument then in principle these are related by

$$|E| \simeq |\delta \cos x| \text{ and } |\epsilon| \simeq \left|\frac{\delta \cos x}{\mathrm{Ci}(x)}\right|.$$

That is accuracy will be limited by ***machine precision*** near the origin and near the zeros of $\cos x$, but near the zeros of $\mathrm{Ci}(x)$ only absolute accuracy can be maintained.

The behaviour of this amplification is shown in Figure 1.

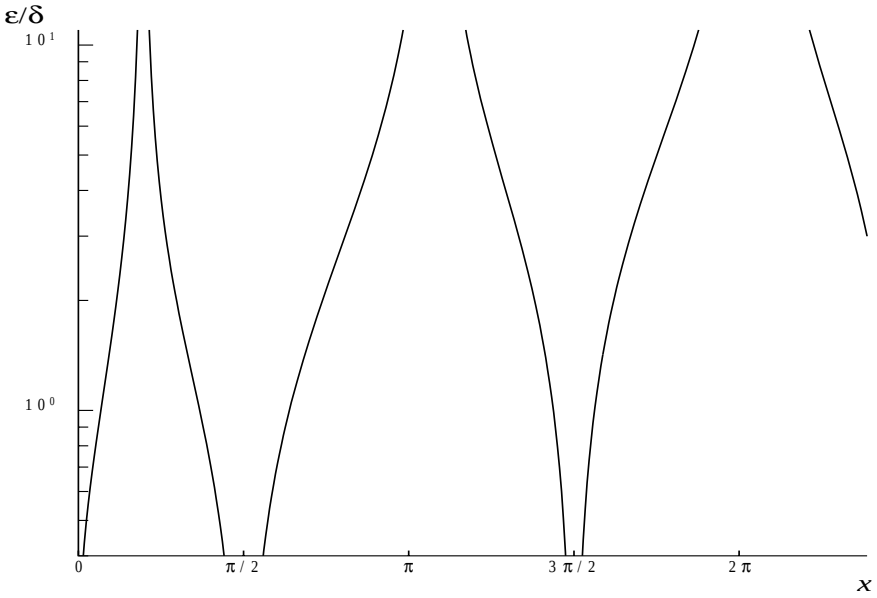

Figure 1

For large values of x, $\mathrm{Ci}(x) \sim \dfrac{\sin x}{x}$ therefore $\epsilon \sim \delta\, x \cot x$ and since δ is limited by the finite precision of the machine it becomes impossible to return results which have any relative accuracy. That is, when $x \geq 1/\delta$ we have that $|\mathrm{Ci}(x)| \leq 1/x \sim E$ and hence is not significantly different from zero.

Hence x_{hi} is chosen such that for values of $x \geq x_{hi}$, $\mathrm{Ci}(x)$ in principle would have values less than the ***machine precision*** and so is essentially zero.

8 Further Comments

None.

9 Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     S13ACF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      real             S13ACF
      EXTERNAL         S13ACF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S13ACF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y        IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S13ACF(X,IFAIL)
*
      WRITE (NOUT,99999) X, Y, IFAIL
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,1P,2e12.3,I7)
      END
```

9.2 Program Data

```
S13ACF Example Program Data
             0.2
             0.4
             0.6
             0.8
             1.0
```

9.3 Program Results

```
 S13ACF Example Program Results

      X           Y        IFAIL

    2.000E-01  -1.042E+00      0
    4.000E-01  -3.788E-01      0
    6.000E-01  -2.227E-02      0
    8.000E-01   1.983E-01      0
    1.000E+00   3.374E-01      0
```